Increased Stability Limits of the Turning Process by Intentionally Delayed Force Control

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Summary: The efficiency of the turning process is limited mainly by the chatter vibrations. The source of these vibrations is the surface regeneration effect. The efficient technological parameters can be selected from the computed stability chart. Even if the computation is perfect, there is a need to increase the stable region. In this work, two different intentionally delayed control method of the turning process were analyzed based on the characteristic equation of the governing equation. It was found, that the control with proper point delay and with the distributed delay can increase the stability limits even in case of uncertain technological parameters.

Keywords: Stability, Control, Turning, Feedback.

1. Introduction

The efficiency of the turning process is limited mainly by the chatter vibrations [1]. The source of these vibrations is the surface regeneration effect, which can be modelled by time delayed differential equations [2]. The stability theory of the time delayed systems and their numerical implementation is well described in the literature [1,3,4]. The efficient technological parameters can be selected from the computed stability chart, however, the results are unreliable in some cases when the modal parameters (like natural frequency and damping ratio) and the cutting coefficients have an uncertain value. Even if the computation is perfect, there is a need to increase the stable region.

In this work, a new method is shown where delayed control force is applied. The digital force controls have a small time delay due to the computational time of the control unit. Even this small time delay can be relatively large at higher spindle speeds, which could also cause stability problems. In the following, different intentionally delayed control techniques are analysed with single point-delay and with distributed delay, too.

2. Mechanical model

The turning process is modelled by the commonly used orthogonal cutting process [5] (see Fig. 1). If the vibration of the tool is considered, then the instantaneous chip thickness h is defined by

$$h(t) = h_0 + x(t) - x(t-\tau) , \qquad (1)$$

where h_0 is the static component of the chip thickness created by the feed motion. The dynamic chip thickness component $x(t) - x(t-\tau)$ consists of the current tool position and the delayed one which is generated by the so-called surface regeneration effect [1,2]. The time delay is equal to the time period of the spindle rotation $\tau = 2\pi/\Omega$, where Ω is the spindle speed. The cutting force is modelled by a linear function:

$$F(t) = k_1 w h , \qquad (2)$$

where the w is the chip width and k_1 is the cutting coefficient. An external force control based on the measured tool position is also considered. The governing equation of this system is given by

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = k_1 w\big(h_0 - x(t) + x(t-\tau)\big) + Q(x_t) , \qquad (3)$$

where Q is the controlled external force, m, c and k is the modal mass, damping, and stiffness of the system, respectively. The perturbed motion around the stationer position ($x_{st} \approx w h_0 k_1 / k$) is described by variable $u(t)$. The stability of the stationer position can be analyzed through governing equation of $u(t)$:

$$m\ddot{u}(t) + c\dot{u}(t) + (k + k_1 w)u(t) = k_1 w u(t-\tau) + Q(u_t) . \qquad (4)$$

For the further analysis, the dimension less form of Eq. (4) is used:

$$u''(\theta) + 2\xi u'(\theta) + (1+\tilde{w})u(\theta) = \tilde{w}u(\theta - \tilde{\tau}) + q(u_\theta) , \qquad (5)$$

where the dimensionless time is $\theta = \omega_n t$, the dimensionless time delay is $\tilde{\tau} = 2\pi\omega_n/\Omega$, the damping ratio is $\xi = c/(2m\omega_n)$, the natural frequency is $\omega_n = \sqrt{k/m}$, the dimensionless chip width is $\tilde{w} = k_1 w/(m\omega_n{}^2)$ and the dimensionless control force is $q(u_\theta) = Q(u_t)/k$.

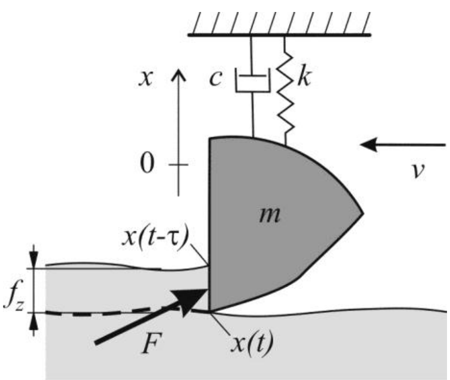

Figure 1. Mechanical model of the orthogonal turning.

The most commonly used PD controller could be used for the force control Q to improve the stability by increasing the stiffness and the damping of the system, however, the small essential time delay of the control force could lead to further stability problems [6]. If the time delay of the control cannot be circumvented, then we may use it against the stability.

In the next section, the stability of the system is analyzed in case of different intentionally delayed control methods.

2. Control with point delay

The source of the stability problem is the regenerative effect. The delayed term could be cancelled out by the control force, if $q(u_\theta)=\tilde{w}u(\theta-\tilde{\tau})$, thus, the system would be stable for any combination of technological parameters. In a real case situation the measured cutting coefficient k_1 and the time delay τ contain some error, so the control force would have the following form:

$$q(u_\theta)=-(1+\eta_{k_1})\tilde{w}u(\theta-\tilde{\tau}(1+\eta_\tau)), \tag{6}$$

where η_{k_1} and η_τ are the relative errors of the measurement of k_1 and τ (or Ω), respectively. The stability of the controlled system is analyzed by the characteristic equation, after substituting the trial function $q(t)=Ae^{\lambda t}$ into the combination of Eqs. (5) and (6):

$$\lambda^2+2\xi\lambda+(1+\tilde{w})=+\tilde{w}e^{-\lambda\tilde{\tau}}-(1+\eta_{k_1})\tilde{w}e^{-\lambda\tilde{\tau}(1+\eta_\tau)}. \tag{7}$$

The linearization of Eq. (6) according the errors parameters gives

$$\lambda^2+2\xi\lambda+(1+\tilde{w})=\tilde{w}e^{-\lambda\tilde{\tau}}(\lambda\tilde{\tau}\eta_\tau-\eta_{k_1}), \tag{8}$$

which can be also derived from an oscillator with a delayed PD control $q(u_\theta)=Pu(\theta-\tilde{\tau})+Du'(\theta-\tilde{\tau})$, where the proportional gain is $P=-\tilde{w}\eta_{k_1}$ and the differential gain is $D=\eta_\tau\tilde{\tau}w$.

At the stability limit $\lambda=i\omega_c$, where ω_c is the so-called chatter frequency. The stability boundaries are determined based on the D subdivision method [1]. Efficient computations of the charts are performed by using the multi-dimensional bisection method [7].

2.1. Stability chart for control with point delay

Usually the spindle speed can be measured with high accuracy, but the accuracy of the cutting coefficient is much smaller due to the influence of the material properties, tool geometry, tool wear etc. Therefore, the stability charts are plotted for different parameters η_τ=-1,-0.5,0,0.5,1% and η_{k_1}=-10,-5,0,5,10%. Figure 2 shows that the stability boundaries are not sensitive to the error of cutting coefficient. Even in case of $\eta_\tau=0$ and $\eta_{k_1}=20\%$, the lower envelope of the stability boundary curves still ~7 times higher than the one without control.

The influence of the error of the time delay η_τ is much stronger, however, in case of an easily achievable precision ($\eta_\tau=\sim 1\%$), the stability limits decrease only in the lower spindle speed range.

High chip width and high spindle speed is achievable by using this control method, which leads to an efficient turning process with large material removal rate.
In some situation the lower spindle speed range is also important, like machining titanium, where a different control technique could give better results.

3. Control with distributed delay

To increase the stability limits in the lower spindle seed range, the new control method were inspired by the process damping phenomena [1,8]. The short delay [1] model gives an accurate model of the process damping, where a cutting force is modelled by distributed delay in the following form

$$F=k_1w\int_0^{\tau_{\max}}\gamma(\phi)h(t-\phi)\mathrm{d}\phi, \tag{9}$$

where $\gamma(\phi)$ is the weight function of the distributed delay terms, which is usually approximated by a segment of a trigonometric function or by an exponentially decaying function. For the ease of the programming of the control unit, an exponential function is defined

$$q(u_\theta)=P\sigma\int_0^\infty e^{-\sigma\phi}u(\theta-\phi)\mathrm{d}\phi, \tag{10}$$

where σ is the kinetic constant of the weight function. The characteristic equation of the system is given by

$$\lambda^2+2\xi\lambda+(1+\tilde{w})=+\tilde{w}e^{-\lambda\tilde{\tau}}+P\sigma\int_0^\infty e^{-\sigma\phi}e^{-\lambda\phi}\mathrm{d}\phi. \tag{11}$$

The integral of the right hand side can be simplified, and rearranged as follows:

$$\begin{aligned}&\lambda^3+(\sigma+2\xi)\lambda^2+(2\xi\sigma+1+\tilde{w})\lambda+\\&\qquad\sigma(1+\tilde{w}-P)-(\lambda+\sigma)\tilde{w}e^{-\lambda\tilde{\tau}}=0.\end{aligned} \tag{12}$$

Equation (12) shows, that the exponential term transforms the system into a third order delayed differential equation with delayed position and delayed velocity.

3.1. This Stability chart for control with point delay

The computed stability maps for different control parameters are shown in Fig. 3. It is important to note, that the envelope of the stability curves increases uniformly at every spindle speeds for every positive P and σ parameters. Other advantage of this control is that there is no need for measuring the spindle speed or the cutting coefficient.

4. Conclusion

Two different intentionally delayed control methods of the turning process were analyzed based on the characteristic equation of the governing equation. It was found, that the control with point delay could be used in the high speed range even in case of measurement errors of the spindle speed and the cutting coefficient. Control with distributed delay can be used at every spindle speeds, and all the positive parameters of the gain P and the kinetic constant σ of the programmed exponential weight function increase the stable region.

Our further plan is to optimize the control function and create intelligent adaptive control method.

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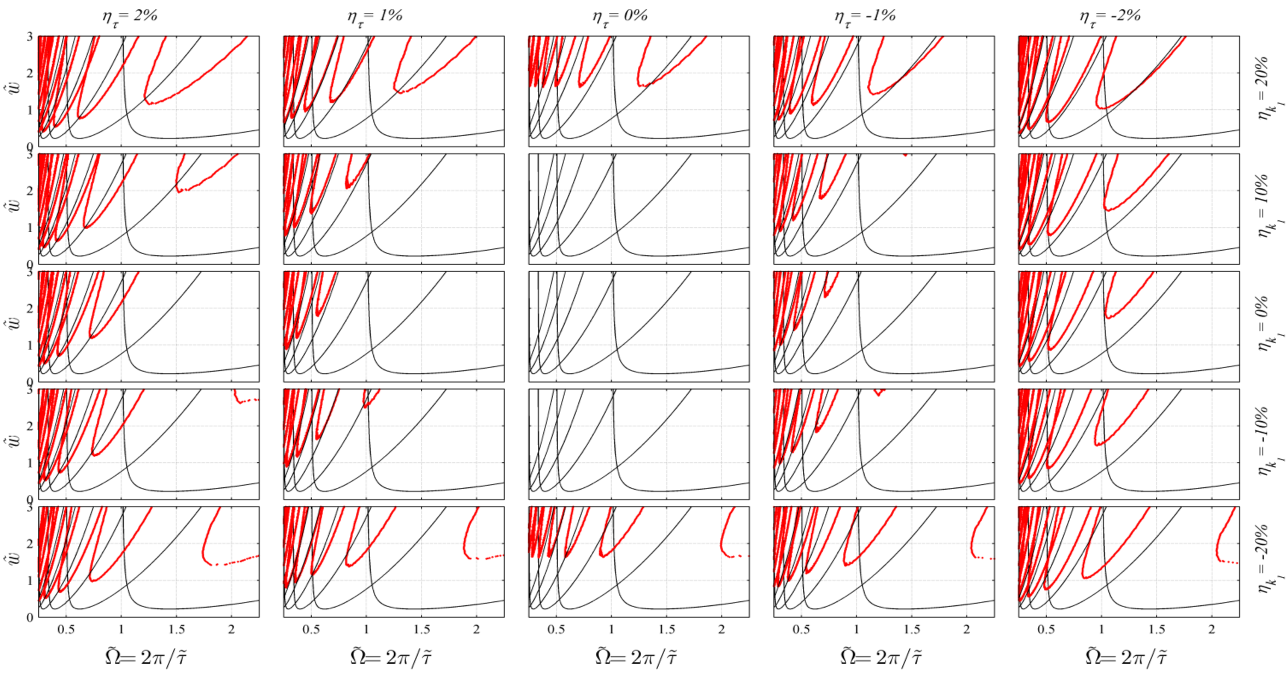

Figure 2. Stability boundaries of the turning process (red thick lines: control with point delay, black thin line: no control).

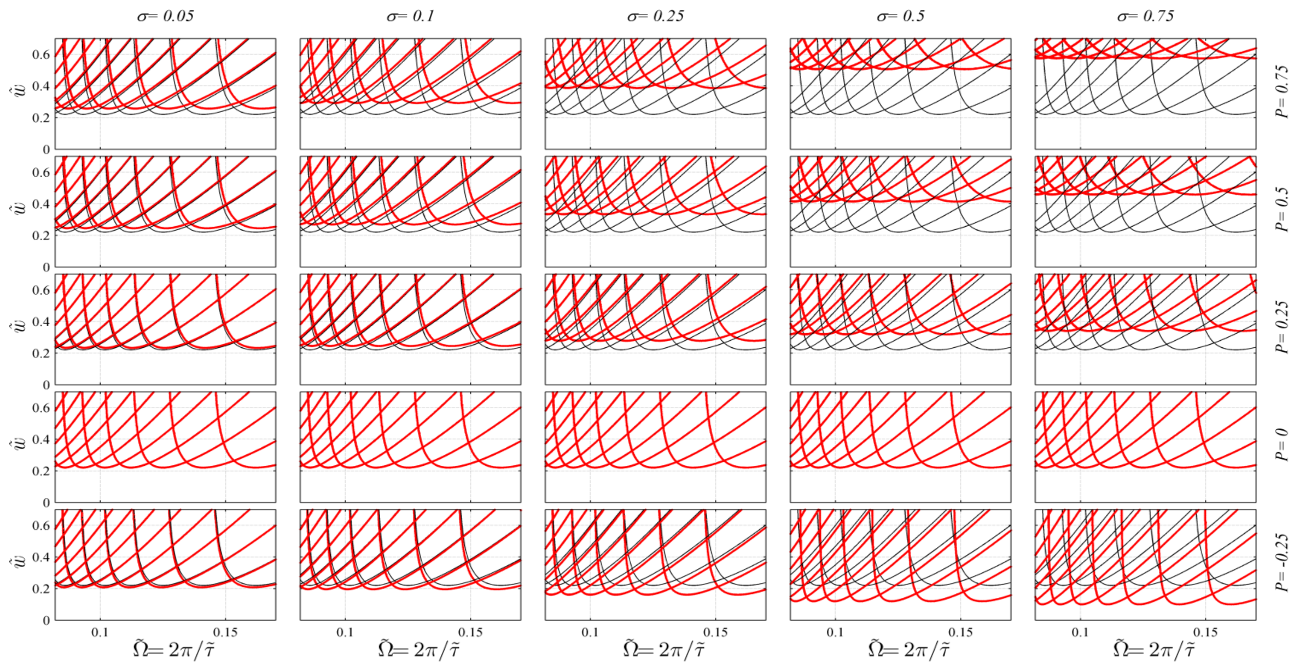

Figure 3. Stability boundaries of the turning process (red thick lines: control with distibuted delay, black thin line: no control).