Robust Pareto Points with Respect to Crosswind of an Active Suspension System

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Summary: In this work, the optimization of control parameters for an active suspension system is considered. Here, two objectives – energy and comfort – play an important role. Thus, a multiobjective optimization problem is formulated which can be solved numerically with a set-oriented approach, for example. The result is the set of optimal compromises of the objectives, the so-called Pareto set. In case of the active suspension system, the crosswind has an influence on the system and thus also on the Pareto set. Therefore, crosswind is modelled as an external parameter which leads to a parametric multiobjective optimization problem. In this work we investigate the question which Pareto points are robust with respect to crosswind variations. We use a recently developed algorithm for the computation of robust solutions. Here, a classical variational problem is formulated which describes a parameter-dependent solution path that varies as little as possible in parameter space. Making use of necessary conditions for this variational problem a nonlinear system of equations is formulated and solved numerically. Although one can observe significant variations of the Pareto sets for our application example, two robust Pareto points are computed. Hence, we can show that our approach reveals additional information which might be used in the self-optimization process in the future.

Keywords: Optimization, Robustness, Control, Mechatronic.

1. Introduction

In almost any application optimization plays an important role. In a variety of these applications not only one objective but several ones are desired to be optimal at the same time. For instance, in manufacturing cost has to be minimized, but at the same time also quality is desired to be maximal - at least to a certain degree. The development of theory and algorithms for the determination of solutions that are as good as possible with respect to all objectives is the task of multiobjective optimization (cf. e. g. [1]). The example mentioned above already illustrates that the several objectives typically contradict each other and thus do not have identical optima. Consequently, the solution of a multiobjective optimization problem is given by the set of optimal compromises of the objectives, the so-called Pareto set. In the case of minimization problems the Pareto set is given by the set of solutions in which the value of any objective function can only be decreased at the cost of increasing another one.

If the objective functions do not only depend on the optimization parameters but also on external parameters which are not desired to be optimal but vary within known intervals we have a parametric multiobjective optimization problem. Imagine for instance the design process of a car which is desired to be optimal with respect to comfort (first objective) and safety (second objective). On the road, the car will be caught in a cross-fire of influences, like cross wind, wet roads, or changes in temperature. The only information one has is that these influences can be estimated to lie in a certain interval, probably given by the weather forecast. Mathematically, this leads in the simplest case to the parametric multiobjective optimization problem

$$\min_{p} F(p,\lambda) \text{ with } F : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^k. \qquad (1)$$

Thus the solution (the Pareto set) also depends on the parameter λ. For the designer of a car those Pareto points are desirable, where the Pareto points or their images under F change as little as possible while the parameter varies. These points are called λ-robust Pareto points.

For the development of modern, innovative mechatronic systems, the concept of self-optimization has been introduced (cf. [2]). A technical system is self-optimizing whenever three steps are repeated iteratively during operation: the analyis of the current situation (step 1), the determination of the system's objectives (step 2) and the adaptation of the system's behavior (step 3). Thus, self-optimization goes beyond the classical adaptive control in which the system's objectives are not changing during runtime. Multiobjective optimization plays an important role in the self-optimization process. One possibility to realize Step 3 - for a technical system which can be described by a mathematical model - is to choose different solutions during operation time from a previously approximated Pareto set. For this choice, the robustness of Pareto points with respect to variations of external parameters is an important information. Our robustness studies can help the decision maker to formulate adequate heuristics for the choice of Pareto points during operation time.

In this work, we consider an active suspension system in which both energy and comfort are desired to be optimal. Cross wind has a significant influence on the Pareto optimal solutions. Thus, we investigate the computation of robust Pareto points with respect to crosswind variations in the following.

2. Application Example: Active Suspension System

A new, innovative traffic and transport concept has been developed at the University of Paderborn [3]. Its basic idea is to use the existing railway infrastructure with small autonomously driven vehicles, called RailCabs. These RailCabs accelerate by

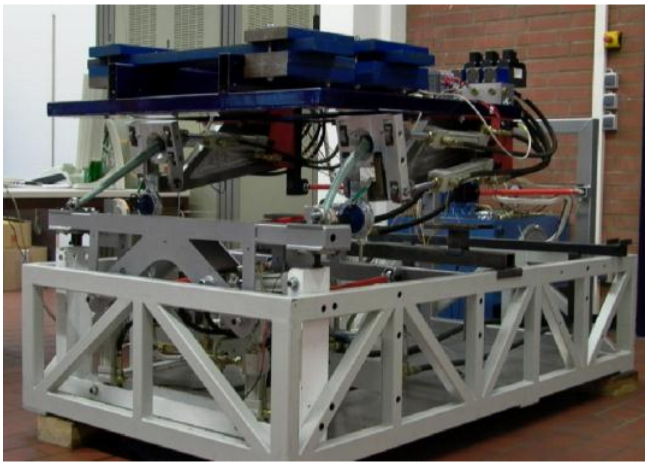

Figure 1. The test rig for the active suspension system.

means of a doubly fed asynchronous linear motor set between the existing tracks and are equipped with an active guidance as well as an active suspension system.

The active suspension system performs the task of compensating for bumps and other excitations of the railway in order to increase passenger comfort in vertical and lateral directions. To design the controller there is a Hardware-in-the-Loop (HiL) test rig which emulates the active suspension system of a RailCab, see Fig. 1. A model of this test rig serves as an application example in this contribution.

The test rig consists of a coach body which can move in vertical, horizontal and rotational (body roll) degrees of freedom. Beneath the coach body there are two symmetrically mounted actuator groups, each one consisting of a guide kinematics, which is connected to a GRP (glasfiber reinforced polymers) spring and three hydraulic cylinders. The main function of the actuator groups is to exert forces on the coach body by deflecting the GRP springs. A chassis framework that can again be displaced by three hydraulic cylinders is used to simulate the railway excitation.

In this contribution a simple sky-hook controller is used. It depends on three controller parameters $p = \{p_1, p_2, p_3\}$ representing the damping of each degree of freedom of the coach body. The multiobjective optimization described subsequently is used to compute the optimal controller configurations with respect to the two contradicting objectives comfort and energy consumption. Both of them are nonlinear functions

$$f_{1,2} : \mathbb{R}^3 \to \mathbb{R},\ p \mapsto f_{1,2}(p) = \frac{1}{T}\int_0^T y(t)^T Q_{1,2}\, y(t)\mathrm{d}t. \qquad (2)$$

They depend on the response *y(t)* of a linear system that is weighted by a diagonal positive semi definite matrix $Q_{1,2}$. The linear system

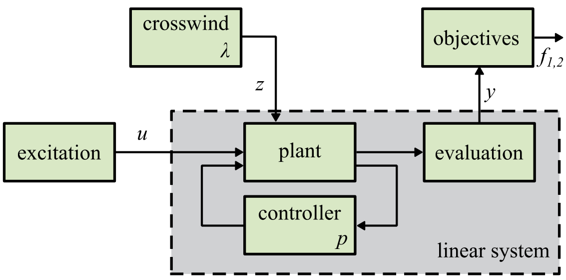

Figure 2. Optimization model of the active suspension system.

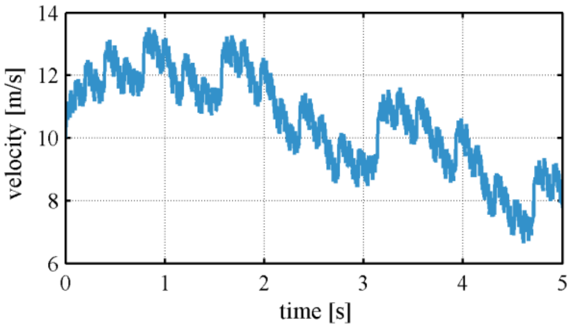

Figure 3. One example crosswind velocity profile.

$$\dot{x}(t) = A(p)x(t) + Bu(t) + Ez(t),$$
$$y(t) = Cx(t) \qquad (3)$$

itself depends on the optimisation parameters p and is used to compute the response to a fixed excitation $u(t)$ for a constant simulation time T. The crosswind is modelled as an additional disturbance $z(t)$. The system consists of several parts as shown in Fig. 2. On the one hand there are the plant and the controller which describe the dynamic behaviour of the test rig. On the other hand the excitation and evaluation models define the optimization problem. The excitation model generates a stochastic excitation profile and the evaluation model consists of several low-pass and band-pass filters, see [4] for more details. The last part is the crosswind-model which is used to compute a parameter-dependent crosswind profile that acts as an additional disturbance to the plant. The parameter λ represents the mean velocity of a periodic wind velocity profile. A sketch of the wind profile can be found in Fig. 3.

3. Multiobjective Optimization, Robust Pareto Points and Numerical Results

As mentioned above, comfort and energy are desired to be optimal at the same time in this application. Mathematically speaking, this leads to a *multiobjective optimization problem*

$$\min_p F(p),\ F = (f_1(p),\ f_2(p))^T,\ p \in \mathbb{R}^3. \qquad (4)$$

Here, the minimization refers to the comparison of vectors. A vector is less or equal than another vector, if all its entries are less or equal than the entries of the other vector.

In contrast to single objective optimization, where typically the solution is given by a single minimum of one objective function, in multiobjective optimization one ends up with a set of optimal compromises of solutions, the so-called *Pareto set*. Using a set-oriented approach for the numerical approximation of the entire Pareto set (cf. [5]) we obtained the sets plotted in Figure 4 for three specific λ-values (here, smoothing splines have been used to interpolate between the numerical approximation of the Pareto set). Recently, numerical algorithms have been developed which allow for the computation of robust Pareto points, i.e. Pareto points which change as little as possible under the variation of an external parameter (cf. [6], [7]). In the application under consideration, the crosswind λ is such an external parameter. To compute Pareto points that stay constant under the variation of crosswind, the variational problem

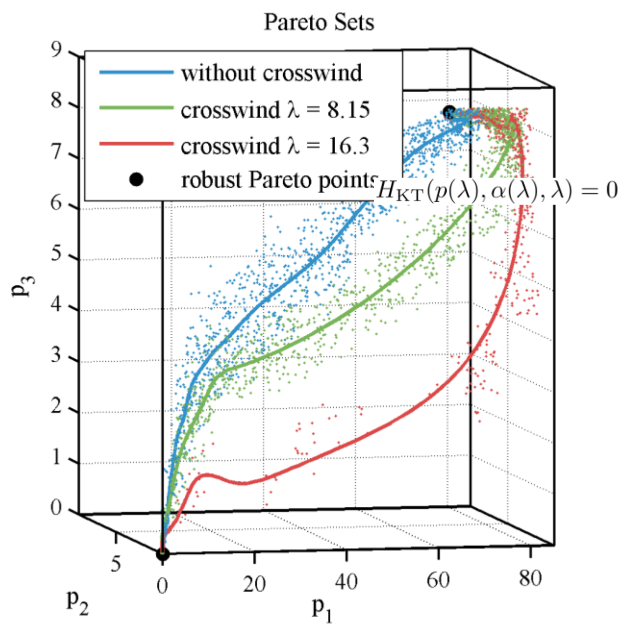

Figure 4. Pareto sets for three specific crosswind values and robust Pareto points.

$$\begin{aligned} &\min_{(p(\lambda),\alpha(\lambda))} \int_{\lambda_{\mathrm{start}}}^{\lambda_{\mathrm{end}}} \|p'(\lambda)\|_2^2 \, d\lambda \\ &\text{s. t. } H_{\mathrm{KT}}(p(\lambda), \alpha(\lambda), \lambda) = 0 \end{aligned} \tag{5}$$

is considered. Here, the constraint stems from a necessary condition for Pareto optimality, the so-called *Kuhn-Tucker equations*, which state that for each value of λ there exists a weight vector α(λ) in a Pareto point *p(λ)* such that the following equations are satisfied (in our case for $k=2$):

$$H_{\mathrm{KT}}(p(\lambda), \alpha(\lambda), \lambda) = \begin{pmatrix} \sum_{i=1}^{k} (\alpha_i(\lambda))^2 \, \nabla_p f_i(p(\lambda), \lambda) \\ \sum_{i=1}^{k} (\alpha_i(\lambda))^2 - 1 \end{pmatrix} = 0 \tag{6}$$

Although this is only a necessary condition, numerical algorithms typically make use of this criterion. The integral in (5) means that the energy of the λ-dependent curve of Kuhn-Tucker points is minimized. If points exist in which all Pareto sets intersect, this is the same as the minimization of the curve length.

A necessary condition for optimality of (5) is given by the so-called *Euler-Lagrange equations* (cf. e. g. [8]). In [6,7] we have considered a discrete formulation of the Euler-Lagrange equations going back to [9] which leads to a system of nonlinear equations that characterizes candidates for robust Pareto points. This system of equations is given as

$$\begin{aligned} H_{\mathrm{KT}}(p_j, \alpha_j, \lambda_j) &= 0 \;\; \forall\, j = \\ \left(\mu_j^T \cdot \frac{\partial}{\partial \alpha_j} H_{\mathrm{KT}}(p_j, \alpha_j, \lambda_j)\right)^T &= 0 \;\; \forall\, j = \\ \left(\frac{p_{j+1} - 2p_j + p_{j-1}}{h^2}\right) - \left(\mu_j^T \cdot \frac{\partial}{\partial p_j} H_{\mathrm{KT}}(p_j, \alpha_j, \lambda_j)\right)^T &= 0 \;\; \forall\, j = \\ \left(\frac{p_1 - p_0}{h^2}\right) - \frac{1}{2}\left(\mu_0^T \cdot \frac{\partial}{\partial p_0} H_{\mathrm{KT}}(p_0, \alpha_0, \lambda_{\mathrm{start}})\right)^T &= 0 \\ -\left(\frac{p_N - p_{N-1}}{h^2}\right) - \frac{1}{2}\left(\mu_N^T \cdot \frac{\partial}{\partial p_N} H_{\mathrm{KT}}(p_N, \alpha_N, \lambda_{\mathrm{end}})\right)^T &= 0 \end{aligned} \tag{7}$$

where N+1 is the number of discretization points on the curves p(λ), α(λ) and μ(λ), which itself is a so-called curve of Lagrangian multipliers. We chose N=10 discretization points of the crosswind λ between 0 m/s and 16.3 m/s. According to this choice, the values of λ_j are given by $\lambda_j = 0 + j \cdot 0.163$, $j=0,\ldots,N$. The system of equations consists of 99 equations in 99 unknowns. For the computation of robust points in case of the active suspension system we have used the MATLAB solver *fsolve* [10] to compute solutions numerically. The results are given in Figure 4 (black dots).

There are two robust Pareto points which is an interesting result, especially in view of the significant variations of the entire Pareto sets. The lower-left robust point in Fig. 4 could be expected from the engineering as well as the mathematical point of view. At this edge of the Pareto set the configurations are energy-optimal. In the case of our active suspension system energy-optimality stands for a passive system without any active suspension, i.e., $p_1=p_2=p_3=0$. This is independent of the crosswind values. The fact that our algorithm computes this robust point enhances its suitability even for complex mechatronic systems.

More interesting is the second robust point, which is located at the opposite edge of the Pareto set. There is not a simple reason, neither mathematically, nor from the engineering point of view, which indicates that in advance. Hence, this is really some additional information about the active suspension system which might be used in the self-optimization process in future work. The availability of robust Pareto points introduces a classification of optimal system configurations that can be used during system operation, for example if dependability is one of the design objectives.

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